# The Mathematical Theory Behind Casting Nines

Casting nines works because of the mathematical theory of residues and the fact that our numeration system is based on ten.

The theory of residues explains even and odd numbers. A number is even if there is no remainder (residue) when it is divided by two. It is odd if there is a remainder of one when divided by two. Algebraically, this can be expressed as follows:

| | | |
|---|---|---|
| Even number | = | *2m* |
| Odd number | = | *2m + 1* |

Upon division by three, there are three possible residue classes:

| | |
|---|---|
| *3m* | (a multiple of three) |
| *3m + 1* | (one more than a multiple of three) |
| *3m + 2* | (two more than a multiple of three) |

With the divisor nine, there are nine possible residue classes (remainders):

| | | |
|---|---|---|
| *9m* | *9m + 3* | *9m + 6* |
| *9m + 1* | *9m + 4* | *9m + 7* |
| *9m + 2* | *9m + 5* | *9m + 8* |

Every number falls into a certain residue class, and in each class there are many numbers. The chart on p. 6 lists numbers from 0 – 998 in columns by residue class. The residue class of a number is an unchangeable part of its character.

The three important laws of residues are listed below. These are demonstrated in simple terms in the worksheets in this booklet.

— The residue of a number is equal to the sum of the individual digits in the number. If this sum exceeds eight, we replace it by its residue.

— The residue of the sum of two numbers is equal to the sum of the residues of the individual numbers, with the proviso that if the last sum is greater than eight, we use its residue instead.

— The residue of the product of two numbers is equal to the product of the residues of the individual numbers, with the proviso that if the last product is greater than eight, we take its residue instead.

If a sum or product is correct, the residues *must* match. If there is no match, the answer is definitely wrong. If there *is* a match, there is still a chance that the answer may be in error. This can happen when numbers are transposed, when a zero is added or omitted, or whenever both numbers fall into the same residue class for any reason. Casting nines will alert students to *most* but not *all* computational errors. It is not a perfect test. Its great usefulness lies in its speed and ease of use.